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RELATIONS BETWEEN NEAR- RINGS AND RINGS

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ABSTRACT

In this paper we investigate the following conditions .

(i) $xy = (xy)^{n(x,y)}$

(ii) $xy = (xy)^{n(x,y)}$

(iii) $xy = y^{m(x,y)} x^{n(x,y)}$

(iv) $xy = xy^{n(x,y)} x$

(v) $xy = x^{n(x,y)} y^{m(x,y)}$ and finally prove that under appropriate additional hypothesis a $d-g$ near ring must be commutative ring. The Theorem proved here is generalization of many recently established results.

1: **INTRODUCTION:** A near rings R is called periodic if for each x in R there exits distinct positive integer m,n such that $x^m = x^n$. A standing result of of Her stein$[7]$ states that a periodic ring is commutative if its nilpotent elements are central. In order to get the analogue of the result in near rings , Bell $[4]$ established that if R is distributively generated $(d-g)$ near rings with its nilpotent elements lying in the centre , then the set N of nilpotent elements of R forms an ideal , and R/N is periodic then R must be commutative . Now we consider the following properties and notice that a near ring satisfying any one of the following is necessarily periodic.

(I) For each $x, y \in R$ there exists a positive integer $n = n(x,y) > 1$ such that $xy = (xy)^n$

(II) For each $x, y \in R$ there exists a positive integer $n = n(x,y) > 1$ such that $xy = (yx)^n$

(III) For each $x, y \in R$ there exists a positive integer $m = m(x,y)$, $n = n(x,y)$ at least one of them is greater than 1 such that $xy = y^m x^n$

(IV) For each $x, y \in R$ there exists an integer $n = n(x, y) > 1$ such that $xy = xy^n x$

(V) For each $x, y \in R$ there exists an integer $m = m(x, y) > 1$, $n = n(x, y) > 1$ such that $xy = x^n y^m$

Recently commutativity of rings under most of the above mentioned conditions has been investigated

In this paper Ligh [9] has remarked that some conditions implying commutativity in rings might turn a class of near rings into rings. The purpose of this paper is to examine whether some of our conditions may imply that certain near rings are rings

Besides providing a simpler and more attractive proof of result due to Bell [5], our theorem generalizes the results proved in [1],[2],[10].

2: NOTATIONS AND PRELIMINARIES

Throughout the paper R is a left near ring with multiplicative centre Z , and N denotes the set of nilpotent elements of R . An element x of R is said to be distributive if $(y+z)x = yx + zx$ and if all elements are distributive then the near is said to be distributive. A near ring R is distributively generated $(d-g)$ if it contains a multiplicative subsemigroup of distributive elements which generates the additive group R^+ , and a near –ring R will be called strongly distributively generated $(s-d-g)$ if it contains a set of distributive elements whose squares generates R^+ .

An ideal of a near ring R is defined to be normal subgroup I of R^+ such that

(i) $RI \subseteq I$

(ii) $(x+i)y - xy \in I$ for all $x, y \in R$ and $i \in I$

In a $(d-g)$ near ring (ii) may be replaced by $(ii)'$ $IR \subseteq I$. A near ring R is called zero symmetric if $0x = 0$ for all $x \in R$ and zero commutative if $xy = 0$ $\Rightarrow yx = 0$ for all $x, y \in R$.

3: RESULTS

We begin with the following known results which will be used extensively. He proofs of

(I) And (II) are straightforward where as those (III) , (IV) and (V) Can be found in $[6]$.

(I) If R is a zero –commutative near –ring, then $xy = 0 \Rightarrow xry = 0$ for all $r \in R$.

(II) A $d-g$ near –ring is always zero symmetric.

(III) A $d-g$ near-ring R is distributive if and only if R^2 is additively commutative.

(IV) A $d-g$ near ring with unity 1 is a ring if R is distributive or R^+ is commutative.

(V) If N is s two sided ideal in a $d-g$ near ring R , then the elements of the quotient group $R^+ - N$ form a $d-g$ near ring , which will be represented by R/N.

We pause to observe that a $d-g$ near ring R satisfying any of our conditions (1) –(4) is necessarily zero commutative . For example, R satisfy (1) and for a pair of elements $x, y \in R$, $xy = 0$. Hence by virtue of (II),

$yx = (yx)^{n_1 = n(y,x)} = yxyxyx...yx = y(xy)^{n_1 - 1}x = (y0)x = 0$ Now we prove the following Lemma

LEMMA: Let R be a near ring $d-g$ be satisfying one of the following conditions, Then $N \subseteq Z$.

Proof :

(1) Since R is a zero commutative, it follows that if $a \in N$ and x is an arbitrary element of R then ax is nil potent. Thus the nil potent element of R annihilate R on both sides. Hence a is central

(2) As same of the above

(3) Let $a \in N$ and $x \in R$, then there exists integers $m_1 = m(a,x) \geq 1$, and

$n_1 = n(a,x) > 1$ such that $ax = x^{m_1} a^{n_1}$. Now choose $m_2 = m(x^{m_1}, a^{n_1}) \geq 1$, and $n_2 = n(x^{m_1}, a^{n_1})$ >1 such that $x^{m_1} a^{n_1} = a^{n_1 n_2} x^{m_1 m_2}$, and hence

$ax = a^{n_1 n_2} x^{m_1 m_2}$. It is clear that for arbitrary t we have an integers $m_1, m_2, ...m \geq 1$ and $n_1, n_2, ...n_t > 1$ such that $ax = a^{n_1 n_2 ...} x^{m_1 m_2}$ his implies $ax = 0$ and hence a is central. Similarly we can prove (4) and (5)

Theorem: Let R be a $d-g$ near- ring satisfying any one of the above (1) –(5) conditions , then R is commutative .

Corollary 1 : Let R be a $d-g$ near- ring satisfying any one condition (1)-(5) . If $R^2 = R$, then R is commutative ring.

Proof : In view of Theorem a $d-g$ near ring with unity satisfying any one of the conditions (1)- (5) is commutative .Thus for any $x, y, z \in R$, we have

$(y+z)x = x(y+z) = xy + xz = yx + zx$. This implies that R is distributive and hence by (III) R^2 is additively commutative. Now $R^2 = R$ means that R is also additively commutative. Hence R is commutative.

Corollary 2: Let R be a $d-g$ near ring with unity satisfying any one of the conditions (1)-(5) . Then R is a commutative ring .

Proof : Applications of (IV) together with our theorem gives the result.

Corollary 3: Let R be a $d-g$ near ring satisfying any one of the conditions (1)- (5) . Then R is commutative.

Proof: By Theorem, R is a commutative $s-d-g$ near ring in which every elements is distributive and by (III) R^2 is additive. Hence the additive group R^+ of the $s-d-g$ near ring is also commutative and R is a commutative ring.

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